Hale School

Mathematics Specialist

Term 3 2017

Test 6 - Statistical Inference

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Instructions:

- **TIME ALLOWED: 40 Minutes**
- **CAS calculators are allowed**
- **External notes are not allowed**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 6% of the year (school) mark**

Question 1 **(9 marks)**

For a continuous random variable that is uniformly distributed on the interval $a \le X \le b$ the variance is given by $Var(X) = \frac{(b-a)^2}{12}$. A sample of 100 values is taken from a continuous random variable that is uniform on the interval $5 \le X \le 15$.

(a) describe the likely distribution of the sample. (2 marks)

The sample would be distributed uniformly over the interval 5 to 15 like the underlying population distribution

✓ uniform

✓ 5 to 15

(b) describe the sampling distribution of the sample mean (2 marks)

The distribution will be approximately normal $(n > 30)$

mean $= 10$

s.d. $= \frac{10}{\sqrt{12}\sqrt{100}} = \frac{1}{\sqrt{12}}$

$= 0.2887$ (4dp)

✓ normal

✓ mean & s.d.

(c) estimate the probability that the mean of the sample is less than 9.7 (2 marks)

$P(\bar{X} < 9.7) = 0.1493$ (4dp)

✓ $P(\bar{x} < 9.7)$

✓ 0.1493

(d) estimate the probability that when 5 samples of 100 values are taken, exactly 2 of the samples have a mean value less than 9.7. (3 marks)

$P($2 out of 5 samples with $\bar{x} < 9.7)$

$= {}^5C_2 \times 0.1493^2 \times 0.8507^3$

≈ 0.1372 (4dp)

✓ binomial

✓ formula (x, n, p)

✓ answer

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Question 2 **(4 marks)**

For the binomial distribution *bin (n, p)*, the mean value is *np* and the variance is *np(1 - p)*.

A sample of 36 values is taken from a binomial distribution *bin (n, p)*. The sampling distribution of the sample mean has a mean value of 16 and the standard error is $\frac{2\sqrt{15}}{15}$.

Find the values of n and p.

$np = 16$ and $\frac{\sqrt{np(1-p)}}{\sqrt{36}} = \frac{2\sqrt{15}}{15}$ ✓ $np = 16$

✓ $\sigma = \sqrt{npq}$

Solving $np = 16$

and $np(1-p) = \frac{12^2}{15}$ ✓ equation

✓ n, p

gives $n = 40$, $p = 0.4$ (CAS)

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Question 3 **(4 marks)**

A factory produces bags of sugar. To test the distribution of the weights of the bags, the company takes a series of random samples. Each random sample has 50 measurements.
The mean weights of the various samples in kg were:

2.05 , 2.04 , 2.01 , 1.99 , 2.06 , 2.03 , 2.02 , 2.04 , 2.01 , 2.05

Use this information to estimate the mean and standard deviation of the weights of the bags of sugar.

$\bar{X}$ values have a mean of 2.03 ✓ $\bar{x}$ mean

and a standard deviation of 0.0209762 (0.0221108) ✓ $\bar{x}$ s.d.

$\therefore$ Mean weight $= 2.03$ kg ✓ x mean

$\frac{\sigma}{\sqrt{50}} = 0.0209762$

$\Rightarrow \sigma = 0.148$ kg (3dp) ✓ σ value

or $\sigma = 0.156$ kg (if s_x used)

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Question 4 **(9 marks)**

A chicken farmer grows chickens for 12 weeks at which time their masses are distributed normally with a mean of 3.2 kg and a standard deviation of 0.3 kg.

(a) Estimate the probability that a sample of 20 chickens has a total mass greater than 64.1 kg. (3 marks)

$P(\text{Total} > 64.1) = P(\bar{x} > 3.205)$ ✓ $\bar{x} > 3.205$

where $\bar{x} \sim N\left(3.2, \left(\frac{0.3}{\sqrt{20}}\right)^2\right)$ ✓ distribution of $\bar{x}$

$P(\bar{x} > 3.205) = 0.4703$ (4dp) ✓ answer

(b) The farmer wants to sell 12 week old chickens in a quantity large enough so that there is at least a 99% chance that the mean mass is within 0.01kg of the advertised mean value of 3.2kg. How many chickens should he sell in a batch? (3 marks)

$99\% \Rightarrow z = \pm 2.575829$ ✓ z value

$2.575829 \times \frac{0.3}{\sqrt{n}} = 0.01$ ✓ equation

$\Rightarrow n = 5971.406943$

Must have at least 5972 in a batch ✓ answer

(c) The farmer allows one shed of chickens to grow for a further 4 weeks (to 16 weeks old). He collects a sample of 16 chickens and observes that there is a mean mass of 3.8 kg with a standard deviation of 0.4 kg. Find a 95% confidence interval for the mean mass of the chickens after 16 weeks. (3 marks)

95% CI $\Rightarrow z = -1.959964$ ✓ $z = \pm 1.96$

CI is $3.8 \pm 1.959964 \times \frac{0.4}{\sqrt{16}}$ ✓ $\bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$

CI is $[3.604, 3.996]$ ✓ values

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Question 5 **(9 marks)**

The sampling distribution of the mean length of samples of 40 salmon has a mean of 82 cm with a standard error of 1.1 cm. Assume that the lengths of the salmon are normally distributed.

(a) Find the mean length of the salmon population and its associated standard deviation. (2 marks)

Mean $= 82$ cm ✓ Mean

Std dev $= 1.1 \times \sqrt{40} = 6.96$ cm (2dp) ✓ s.d.

(b) Find the sample size required so that the standard deviation of the sampling distribution of the sample means is to be 0.7 cm. (2 marks)

$\frac{\sigma}{\sqrt{n}} = 0.7 \Rightarrow \frac{6.96}{\sqrt{n}} = 0.7$ ✓ equation

$\Rightarrow n = 98.86$

Sample size needs to be 99 ✓ answer

(c) 40 samples of 100 fish are collected;
(i) in how many of the samples would the mean length be greater than 81 cm? (3 marks)

$\bar{x} \sim N\left(82, \left(\frac{6.96}{\sqrt{100}}\right)^2\right)$ ✓ distribution

$P(\bar{x} > 81) = 0.9246$ (4dp) ✓ $P(\bar{x} > 81)$

$40 \times 0.9246 = 36.98$

$\therefore$ 37 of the samples ✓ answer

(ii) how many of the fish collected would have lengths greater than 81 cm? (2 marks)

$X \sim N(82, 6.96^2)$ ✓ 0.5571

$P(X > 81) = 0.5571$ (4dp)

$4000 \times 0.5571 = 2228.49$ ✓ answer

Expect 2228 to be > 81cm

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Question 6 **(6 marks)**

Mr Bausor has installed new solar panels on his roof. The amount of power generated in a day by the panels is measured in kWh (kilowatt hours). It is known that the long run standard deviation of such measurements is 4.0 kWh.

(a) Mr Bausor collects a sample of 20 values and obtains a confidence interval for the mean daily power output of $[31.112, 33.688]$. Find

i) the sample mean (1 mark)

$$\frac{31.112 + 33.688}{2} = 32.4$$ ✓ answer

ii) the level of confidence used. (2 marks)

$$z \times \frac{4}{\sqrt{20}} = 33.688 - 32.4 = 1.288$$

$$\therefore z = 1.44$$ ✓ z value

$\therefore$ 85% confidence level ✓ 85%

NormCDf$(-1.44, 1.44, 1, 0) = 0.85013$

(b) Mr Bausor then decides that a 99% confidence interval would be better. He collects more data and finds the new confidence interval to be $[31.281, 34.319]$.
Find the number of pieces of data in the new sample. (3 marks)

99% CI

$$2.575829 \times \frac{4}{\sqrt{n}} = \frac{34.319 - 31.281}{2}$$ ✓ $z = 2.576$

✓ equation

$$\sqrt{n} = \frac{2.576 \times 4}{1.519}$$

$$n = 46.01$$

✓ answer

There were 46 pieces of data in the sample

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END OF TEST